Forced convection heat transfer in doubly connected ducts

S. C. Solanki*, S. Prakash**, J. S. Saini*** and C. P. Gupta**

Forced convection heat transfer in doubly connected ducts bounded externally by a circle and internally by a regular polygon of various shapes is analysed using a finite element method. Hydrodynamically and thermally developed, steady, laminar flow of a constant property fluid is investigated. An insulated outer tube and constant heat flux at the inner core are considered. Temperature profiles as well as Nusselt numbers are presented. Salient characteristics of the temperature field in such passages are identified. Correlations for the Nusself number with aspect ratio are suggested.

Keywords: *doubly connected ducts, thermally developed temperature field, Nusselt number, friction factor, Reynolds number*

Introduction

This paper is in continuation of an earlier paper[1] in which the characteristics of fluid flow in doubly connected ducts bounded externally by a circle and internally by a regular polygon of various shapes have been reported. In the present paper, the forced convection heat transfer results in such passages are reported. Hydrodynamically and thermally developed, steady, laminar flow of a constant property fluid is investigated. Thermal boundary conditions of the second kind, ie insulated outer tube and constant heat flux at the inner core, are considered. A finite element solution algorithm has been developed for the two-dimensional equations governing the flow and temperature fields. As the earlier paper[1] covers (i) the literature review in detail, (ii) utility of the problem, (iii) method of solution, including the finite element solution algorithm, kinds of elements chosen (Fig 2), and number of elements taken (Fig 2), these are not discussed here.

Cheng and Jamil[2] have obtained the Nusselt number values for H1 type thermal boundary conditions for such passages using the collocation method. Sastry[3] has investigated the fifth kind of thermal boundary condition for an annulus with a square core, with the help of a Schwarz–Neumann alternating method.

Analysis

The annular passage in which flow and heat transfer take place is shown in Fig 1. The inner core is concentric with the outer tube and has sharp corners. For the steady, laminar, hydrodynamically as well as thermally fully developed flow of a constant property Newtonian fluid the governing energy equation in the cylindrical coordinate system is

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \theta^2}=\frac{u}{\alpha}\frac{\partial t}{\partial z} \quad (1)$$

The thermal boundary conditions associated with Eq (1), which are considered in the present paper, are of the Neumann type:

$$-k(\partial t/\partial n_1)=\dot{q}''_{w,1} \text{ at the inner wall} \quad (2)$$

$$-k(\partial t/\partial n_1)=\dot{q}''_{w,2}=0 \text{ at the insulated outer wall} \quad (3)$$

where $\dot{q}''_{w,1}$ and $\dot{q}''_{w,2}$ are the heat fluxes at the inner and the outer walls, which are considered invariant in the axial direction.

For the axially invariant heat fluxes at the boundaries and the corresponding fully developed temperature field, the rate of temperature variation in the axial direction is assumed to be constant everywhere and equal to the rate of bulk mean temperature variation in the axial direction[4]. This implies

$$\frac{\partial t}{\partial z}=\frac{\partial t_m}{\partial z}=\frac{\partial t_{w,1}}{\partial z}=\frac{\partial t_{w,2}}{\partial z}=\text{constant} \quad (4)$$

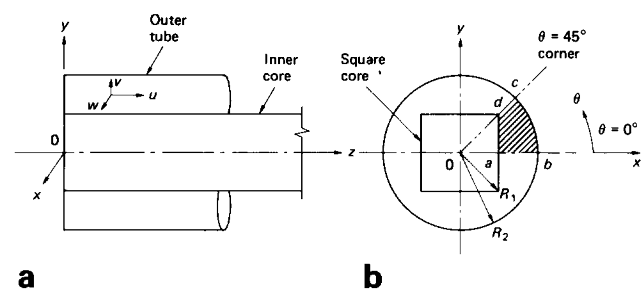

Figure 1 Coordinate system: (a) longitudinal section; (b) cross-section

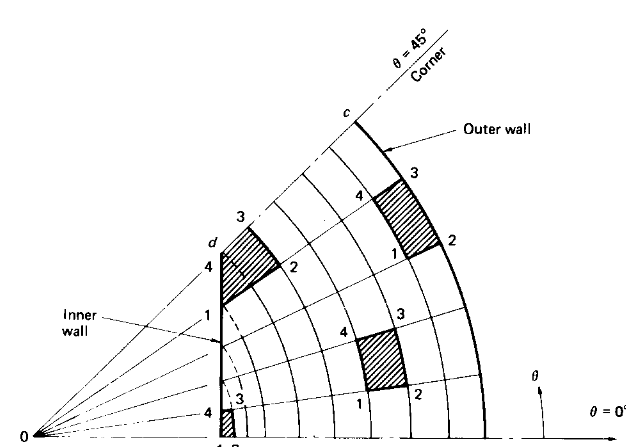

Figure 2 Elements: abcd is smallest symmetrical portion of the cross-section. Shaded subdomains are some typical elements. Total number of elements=145. Total number of nodes=175

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The value of the constant in Eq (4) depends upon the heat fluxes at the inner and the outer walls. From energy balance one can obtain

$$\frac{dt_m}{dz}=\frac{\dot{q}'_{w,1}}{\bar{u}AC_p\rho}=\text{constant} \quad (5)$$

It should be noted that Eq (5) takes into account the fact that the outer wall is insulated. Using Eqs (4) and (5),the energy equation (Eq (1)) is converted into the following dimensionless form:

$$\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}+\frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2}-U^*A^*=0 \quad (6)$$

where

$$U^*=u/\bar{u};\ A^*=R_2P_1/A;\ C_2=\dot{q}'_{w,1}R_2/P_1k \quad (7)$$

and

$$R=r/R_2;\ T=(t-t_{w,2})/C_2;\ n^*=n_1/R_2 \quad (8)$$

Here, it has been assumed that the outer wall, which is circular, is at an average temperature $t_{w,2}$ which is independent of θ. This assumption is expected to be good for most of the range of aspect ratio β. Only as $\beta\rightarrow 1$ may a deviation occur. This situation is discussed below.

The corresponding boundary conditions (Eqs (2) and (3)) for Eq (6) are

$$\partial T/\partial n^*=-1 \quad \text{at the inner wall} \quad (9)$$

$$T=0 \quad \text{at the outer wall} \quad (10)$$

It should be noted that the condition at the outer wall (Eq (10)) implies a known value of temperature $t_{w,2}$ there. The variation of $t_{w,2}$ in the developed region in the axial direction is known through Eq (4). However, the temperature $(t_{w,2})_o$ of the outer wall at the start of the developed region can be determined only after solving the developing region problem, which is more complicated and expected to be three-dimensional in nature. The matching of temperature field for the developing and the developed region makes the temperature field in the developed region unique for the Neumann type of boundary conditions. However, the solution in the developed region is determined here with $(t_{w,2})_o$ as reference temperature. It should be noted that the insulated outer wall condition has been incorporated in the evaluation of dt_m/dz (Eq (5)) and thus in the definition of the dimensionless temperature T through the constant C_2. The temperature distribution in the developed thermal region will, therefore, have a zero gradient normal to the wall at the outer boundary.

Values of velocity, u and U^*, which have been obtained by solving the governing momentum equation for fully developed laminar flow[1], are substituted in Eq (6) when its solution is sought. This implies that the velocity profiles are unaffected by the temperature field since the viscosity of the fluid is assumed to be invariable because of the moderate temperature range under consideration[4,5].

Results and discussion

The results which are presented here are the temperature profiles (Fig 3), isotherms (Fig 4) and the Nusselt numbers (Fig 5 and Table 2).

Confirmatory results

The algorithm developed is tested with a circular annulus case for which the solutions are available[4]. The comparison of both the detailed (not given here, for brevity) and overall results (Table 2) is found to be excellent.

No solution is reported in the literature for the temperature field in the non-circular annulus under consideration for the thermal boundary conditions of the second kind.

Temperature field

Temperature profiles

The typical temperature plots are given for the non-circular annulus with a square core for aspect ratio $\beta=0.5$ in Fig 3, and the corresponding isotherms in Fig 4. Similar trends of the temperature variations are found for other non-circular annuli studied and for higher and lower aspect ratios in each case. Hence, figures for other cases are not given here. It is observed that the temperature difference in the angular direction increases as the aspect ratio increases and as the number of core sides decreases, and vice versa.

Inner wall temperature, $T_{w,1}$

An inspection of Fig 3 reveals that the temperature $T_{w,1}$ at the inner wall is maximum at the midpoint of the core side and it

Notation

A	Area of flow, m^2
C_2	Reference temperature $\equiv \dot{q}'_{w,1}R_2/P_1k$, K
C_p	Specific heat of fluid, J/(kg K)
D_h	Hydraulic diameter $\equiv 4A/P$, m
h	Heat transfer coefficient $\equiv \dot{q}''_{w,1}/(\bar{T}_{w,1}-T_m)$, W/m^2 K
k	Thermal conductivity of fluid, W/(m K)
Nu_{11}	Nusselt number $\equiv hD_h/k$ or $(D_h/R_2)/(\bar{T}_{w,1}-T_m)$
n_1	Unit normal vector (heat flux going to the fluid is positive)
n	Number of inner core sides
P	Wetted perimeter $\equiv P_1+P_2$, m
P_1, P_2	Perimeter of the inner boundary and of the outer boundary, respectively, m
$\dot{q}'_{w,1}$	Constant heat rate per unit axial length from the inner wall, W/m
$\dot{q}''_{w,1}$	$\dot{q}'_{w,1}/P_1$, W/m^2
r, θ, z	Polar coordinates: radial, m; angular, radian; axial, m
R	Dimensionless radial distance $\equiv r/R_2$
R_1, R_2	Radii of the corner of the inner polygonal core and outer circular tube, respectively, m
RN	Dimensionless radial gap $\equiv (R-s\sec\theta)/(1-s\sec\theta)$
s	Perpendicular distance from the centre to the side of the inner core, m
T	Dimensionless temperature $\equiv (t-t_{w,2})/C_2$
T_m	Dimensionless bulk mean temperature $\equiv (\iint Tu\,dA)/(A\bar{u})$
$\bar{T}_{w,1}$	Average dimensionless temperature of the inner wall at an axial location z
t	Temperature, K
$t_{w,1}, t_{w,2}$	Inner and outer wall temperatures, respectively, K
t_m	Bulk mean temperature $\equiv (\iint tu\,dA)/(A\bar{u})$, K
$u, \bar{u}$	Velocity in axial (z) direction and average velocity u at any z, respectively, m/s
α	Thermal diffusivity $\equiv k/\rho C_p$, m^2/s
β	Aspect ratio R_1/R_2
ρ	*Density*

Subscript

s	Inner wall
2	Outer wall

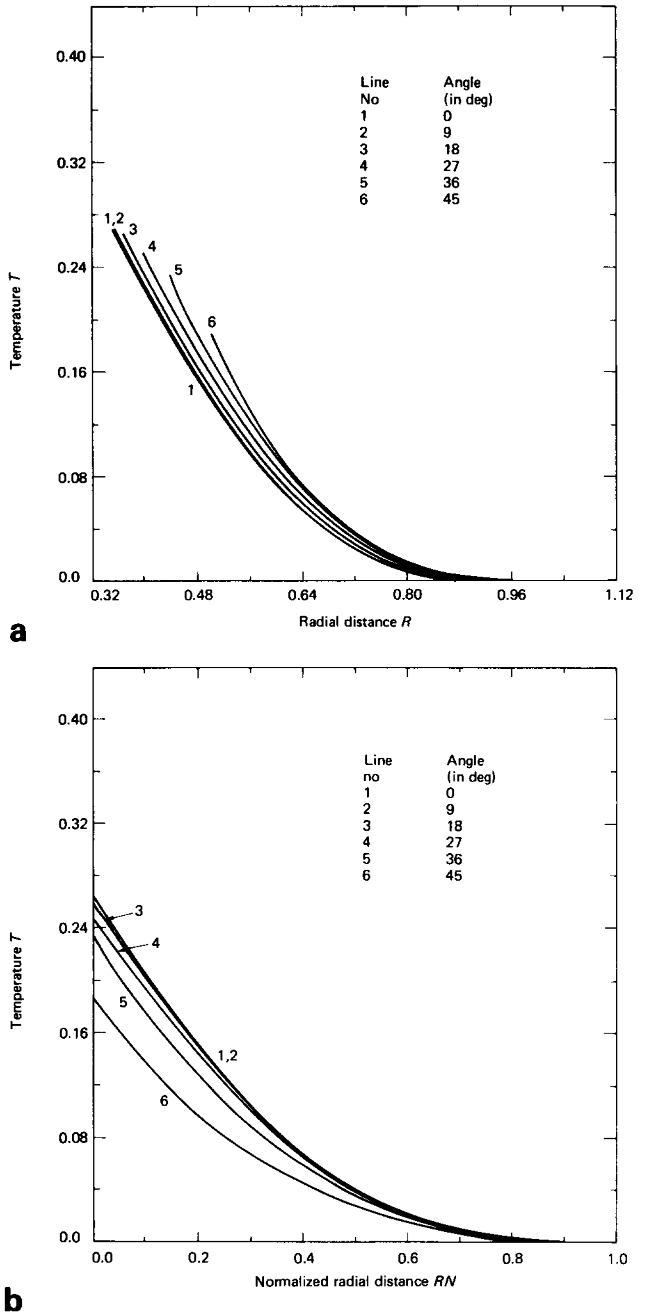

Figure 3 Temperature profiles for annulus with square core, $\beta=0.5$, number of elements=145: (a) T versus R; (b) T versus RN

decreases towards the corner. It is found so for all the cases. It is seen that the difference between the maximum and minimum inner wall temperature decreases as the number of core sides increases, ie the inner wall temperature tends to become uniform as the number of sides increases. For each shape of the inner core, the difference increases with an increase in aspect ratio.

Fluid temperature

Fig 3(a) shows that the fluid temperature T at the same R is highest at the section through the corner and least at the section through the midpoint of the core side, ie it increases with an increase in θ. It is so because, as θ increases, the convection conductance is higher and also points at the same R move closer to the inner wall. The convection conductance is discussed below.

Angular variation of temperature T

The angular variation of temperature is small at low aspect ratios. It is observed that the temperature field in the hydrodynamically and thermally fully developed region could be considered as a function of radial coordinate alone up to a certain aspect ratio dependent on the shape of the inner core, as given in Table 1.

Thus the effect of the inner wall is more pronounced at higher aspect ratios. As the sharpness of a corner of the inner core increases, the angular variation of temperature increases.

Temperature gradient at the outer wall

Fig 3 shows that the temperature gradient normal to the boundary at the outer wall is zero. The outer wall was kept insulated. Thus, it justified the change of the thermal boundary condition at the outer wall from the prescribed temperature gradient given by Eq (3) to the one of prescribed temperature T given by Eq (10).

Convection conductance

It can be observed from Fig 4 that isotherms move closer as θ increases from zero to its value at the corner. This implies that the heat flow rate increases as θ increases. Thus the convection conductance is greatest at the section through the corner.

The relative distances between the isotherms reveal that the temperature gradients near the inner wall are large and those near the outer wall are small. This indicates that the conductance to the heat flow is higher in the region near the inner wall than that in the region near the outer wall.

Nusselt numbers

Fig 5 and Table 2 reveal that Nusselt number Nu_{11} decreases monotonically for the entire range of aspect ratio; the fall is steep when aspect ratio increases from 0.02 to about 0.2, and it is gradual thereafter.

Fig 5 also shows that the Nusselt number does not vary significantly with the shape of the inner core and that it could be predicted for most of the range of aspect ratio from that of the circular annulus having the same aspect ratio. The hydraulic diameter D_h is used in the definition of the Nusselt number.

From Fig 5 it is seen that Nu_{11} and the product $(f\,Re)_1$ of friction factor at the inner wall and the Reynolds number have similar variation. An attempt has been made to relate Nu_{11} to

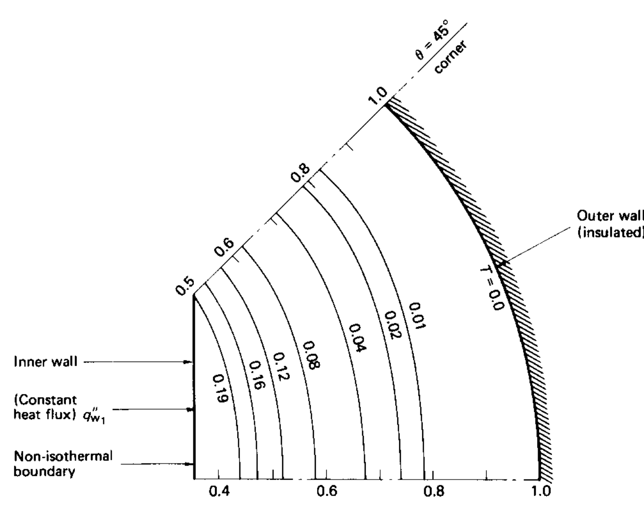

Figure 4 Isotherms for annulus with square core, $\beta=0.5$

Table 1 Maximum aspect ratio for temperature field to be considered a function of radial coordinate alone

Inner core sides, n	3 and 4	6	8	18
Aspect ratio β	0.1	0.3	0.5	0.7

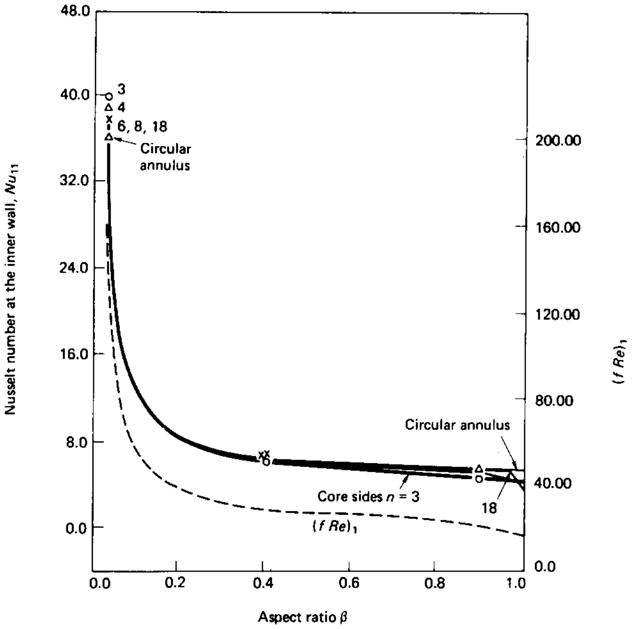

Figure 5 Nusselt number at the inner wall, Nu_{11}, versus aspect ratio β; inner wall heated, outer wall insulated (boundary conditions of the second kind)

Table 2 Nusselt numbers

Aspect ratio β	Core sides, n: 3	4	6	8	18	Circular annulus: Present	Ref 6
0.02	39.880	38.656	37.832	37.570	37.328	35.925	
0.04	23.010	22.458	22.096	21.986	21.884	21.397	
0.05	–	–	–	–	–	–	17.81
0.06	17.182	16.869	16.673	16.617	16.570	16.329	
0.08	14.164	13.977	13.870	13.844	13.826	13.687	
0.10	12.294	12.186	12.136	12.129	12.132	12.044	11.91
0.15	9.686	9.691	9.724	9.747	9.780	9.748	–
0.20	8.311	8.374	8.453	8.493	8.544	8.534	8.499
0.25	7.458	7.554	7.661	7.712	7.776	7.777	7.754
0.30	6.880	6.993	7.118	7.178	7.250	7.258	–
0.35	6.467	6.590	6.723	6.788	6.869	6.881	–
0.40	6.163	6.289	6.424	6.492	6.579	6.594	6.583
0.45	5.933	6.060	6.192	6.261	6.351	6.340	–
0.50	5.754	5.885	6.008	6.076	6.169	6.189	6.181
0.55	5.609	5.749	5.863	5.927	6.020	6.042	–
0.60	5.483	5.641	5.747	5.805	5.895	5.919	5.912
0.65	5.362	5.550	5.656	5.706	5.791	5.815	–
0.70	5.237	5.460	5.582	5.626	5.702	5.726	–
0.75	5.099	5.354	5.516	5.562	5.626	5.650	–
0.80	4.949	5.214	5.435	5.505	5.562	5.584	5.385
0.85	4.793	5.031	5.299	5.426	5.508	5.526	–
0.90	4.643	4.810	5.049	5.248	5.463	5.475	–
0.925	4.574	4.694	4.869	5.076	5.433	5.452	–
0.950	4.511	4.579	4.657	4.822	5.362	5.430	–
0.975	4.454	4.463	4.419	4.479	5.098	5.409	–
0.999	4.405	4.345	4.138	4.023	3.948	5.391	–
1.000	–	–	–	–	–	–	5.385

the aspect ratio so that by knowing just the aspect ratio a designer can predict Nu_{11} for the given annuli. As discussed in the previous paper[1], a cubic equation is found suitable:

$$Nu_{11} = a_2 + b_2\beta + c_2\beta^2 + d_2\beta^3 \tag{11}$$

The least-square curve fit method is employed for evaluating the constants a_2, b_2, c_2 and d_2. The values of the constants are given in Table 3.

Conclusions

For the temperature field in the doubly connected duct geometries studied and for the solution technique used the following conclusions can be drawn.

(1) Application of the present finite element solution algorithm allows successful analysis of the doubly connected ducts studied.
(2) The temperature field can be described by the radial coordinate alone in the hydrodynamically and thermally fully developed region up to a certain aspect ratio dependent on the shape of the inner core (Table 1).
(3) The convection conductance is predominantly high in the region close to the inner wall.
(4) In the corner region, the convection conductance is significantly high.
(5) The hottest spot is the midpoint of the inner core side. (It may be mentioned that in the case of heat transfer in non-circular ducts, a corner attains the highest temperature for axially and peripherally constant heat flux[6].)
(6) The Nusselt number can be expressed as a simple function of aspect ratio (Eq (11)).
(7) The Nusselt number does not vary significantly with the shape of the inner core. It can be predicted closely for most of the range of aspect ratio from the results of the circular annulus having the same aspect ratio. The hydraulic diameter is used in the definition of the Nusselt number.

Acknowledgement

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References

1 Solanki, S. C., Saini, J. S. and Gupta, C. P. Flow through doubly connected ducts. *Int. J. Heat and Fluid Flow*, 1986, **7**, 301
2 Cheng, K. C. and Jamil, M. Laminar flow and heat transfer circular ducts with diametrically opposite flat sides and ducts of multiply connected cross sections. *Can. J. Chem. Eng.* 1970, **48**, 333–334
3 Sastry, U. A. Heat transfer by laminar forced convection in multiply-connected regions. *Acta Technika, Hung.*, 1965, **51**, 181–192
4 Kays, W. M. *Convective Heat and Mass Transfer*, Tata McGraw-Hill, New Delhi, 1975, 106
5 Rohsenow, W. M. and Hartnett, J. P. *Handbook of Heat Transfer, 1st Edn*, McGraw-Hill, New York, 1973
6 Eckert, E. R. G. and Irvine, Jr. T. F. Flow in corners of passages with non-circular cross sections. *Trans. ASME*, 1956, **78**, 709–718

Table 3 Constants in correlation for Nusselt number at the inner wall (Eq (11)) for aspect ratio range 0.1 to 0.9

Core sides	a_2	b_2	c_2	d_2	Stand. dev.
3	16.0168	−51.0596	82.6183	−44.7490	0.041
4	15.8512	−49.8181	81.1283	−44.0850	0.036
6	15.6380	−47.6022	76.5220	−41.0157	0.034
8	15.5008	−46.0784	73.0233	−38.5770	0.035
18	15.3720	−44.6016	69.8659	−36.4954	0.036
Circular annulus (FEM)	15.2320	−43.7787	68.5532	−35.8336	0.035